Allianz

Allianz Research

The anatomy of financial bubbles, crashes & where we stand today

19 July 2022

Eric Barthalon
Head of Capital Markets Research
eric.barthalon@allianz.com

EXECUTIVE SUMMARY

- Some important segments of the US equity market (the NASDAQ, the NASDAQ 100, the S&P 500 home builders index) were most likely in a bubble mode in 2021; they have not yet completed a typical post-bubble correction, offering more downside than upside.
- In contrast, equity markets in other advanced economies as well as in emerging markets were not in bubble territory; they now offer more upside than downside.
- To reach such conclusions, we do not rely on valuation metrics; instead, we use an original methodology that focuses on how past market movements influence investors' psychology in a context of uncertainty.

Is the current market correction a bona fide bubble-bursting episode? And if so, can it get worse?

Impressive as they may be, the 20-30% corrections experienced in the first half of 2022, the worst first-half drop in more than 50 years, have not come in from the cold. They have instead followed a period of rapid price appreciation; their starting points were elevated ones. Hence, two questions are to be answered: firstly, were the starting points of such corrections so high that markets had become bubbles? Secondly, if markets were in a bubble mode, have they deflated as much as they typically do when a bubble bursts? As always, understanding where we are coming from might help to figure out where we are heading to.

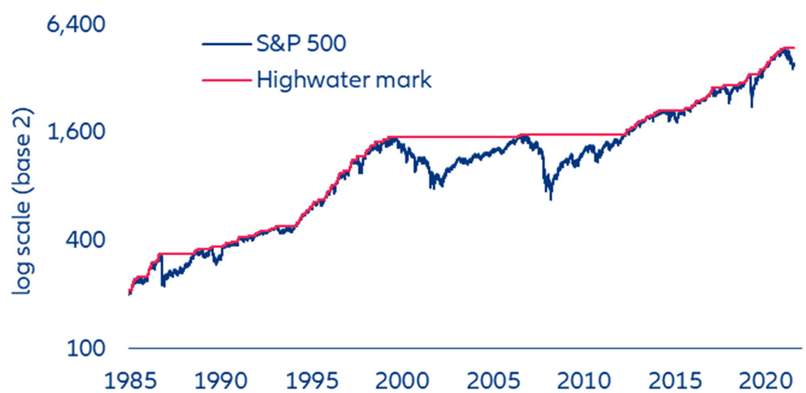
Our answer – namely, yes, some key market segments, mainly in the US, were in a bubble mode, but no, it is not yet time to increase exposure to risky assets – may not be original, but the quantitative method we use to reach it is. As we live in a world of uncertainty ("unknown unknowns") rather than in a world of risk ("known unknowns"), we assume that fundamental valuation is an elusive notion about which investors cannot form rational expectations.[1] We focus instead on how past market movements influence investors' psychology. In our framework, a market becomes a bubble when market participants, looking at the sequence of past returns, perceive it to rise at an elevated pace. To measure such a pace, we use a weighted average of past returns, the weighting factors of which possess a unique property: they are time-varying (or context-dependent). We label this metric the perceived return. To characterize a bubble, we set a threshold of 15% a year for the perceived return.

[1] According to the rational expectations hypothesis (REH henceforth), under risk (known unknowns), people's expectations are nothing but the stochastic forecasts made by "the" relevant economic model. Under uncertainty (known unknowns), people cannot form model-driven ("rational") expectations; more or less quickly, they can only learn from changes in their environment.

The bubble bursts when the market reaches a level that becomes a long-lasting high-water mark (of at least 260 trading days). By combining these two criteria – a perceived return in excess of 15% a year when the bubble bursts and a long-lasting high-water mark – and applying them to 44 different markets (equity markets: 22 in advanced economies, 12 in emerging economies; commodities: six; precious metals: three; crypto-currencies: one) in the post-WWII period, we identify almost 100 bubbles (Table 1).[2] The next and final step of our investigation is to assess whether the paths followed by the perceived returns before as well as after these 100 high-water marks exhibit some common patterns or characteristics. We find they do. We use such patterns and characteristics to assess the current situation in capital markets. According to our methodology, some important segments of the US equity market (the NASDAQ, the NASDAQ 100, the S&P 500 home builders index) were most likely in a bubble mode in 2021, and they have not yet completed a typical post-bubble correction. In contrast, equity markets in other advanced economies as well as in emerging markets were not in a bubble mode.

With the benefit of hindsight, it is easy to spot when a bubble has burst: A bubble bursts when it reaches a long-lasting high-water mark. This provides us with is our first bubble criterion (Figure 1).

Figure 1: High-water marks for the S&P 500 1984 to date.



Sources: Refinitiv, Allianz Research.

When a market is in a rising trend, it frequently reaches new highs and the pattern is one of higher highs (both the blue line and the red one are rising). From time to time, a new high is not exceeded for a long time because it is followed by a protracted price decline (the blue line falls but the red line becomes horizontal because there is no new high coming). Then, if the red line remains horizontal for more than 260 trading days, the last new high becomes what we call a long-standing high-water mark.

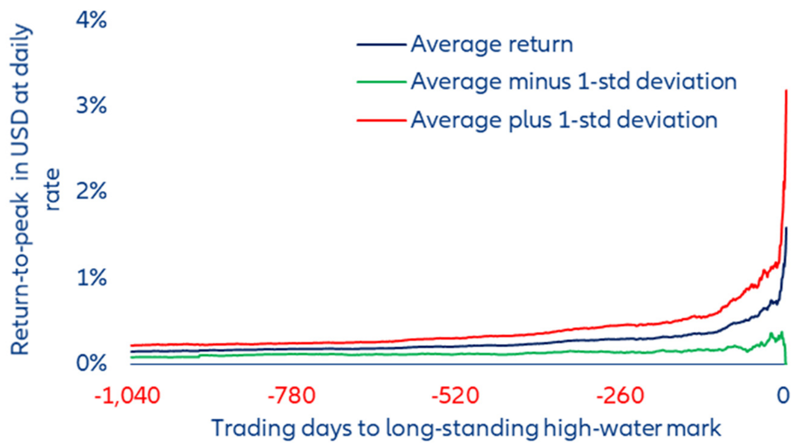
For example, it took the S&P 500 no less than 6,248 trading days (or 25 years) to rise above the level it had reached on 16 September 1929. As for the Nikkei 225, its high-water mark of 27 December 1989 lasted even longer: 8,099 trading days (or 31 years). A long-lasting high-water mark is the first objective criterion that we shall use to define a bubble. But how long-lasting? At least 260 trading days (i.e. one calendar year). Many activities (including investment performance reporting) follow an annual cycle. As any wrong-footed portfolio manager knows, a year is well long enough to generate "underperformance stress" and lose his or her job.

[2] In all cases, we use the longest time series available in Refinitiv. The available timespan is not the same for all assets.

The July 1990, July 1998 and February 2020 episodes provide examples of short-lived high-water marks. In contrast, March 2000 and October 2007 are examples of long-standing high-water marks.

Our second criterion is a perceived rate of return (in USD) higher than 15% a year at the high-water mark. Now that we can date the end of the bubble and assign it the date 0, we can try – looking backward – to characterize what happens during the journey to its terminal burst, as well as – looking forward – what happens in the wake of its burst. Our working assumption is that the pace at which an asset price inflates might indicate how "bubbly" it is. The challenge, then, is to measure that pace, bearing in mind that, in a bubble episode, the sequence of daily returns is not stationary, but typically follows a parabolic path (Figure 2).

Figure 2: The parabolic path of daily return in a bubble (average of 100 bubbles)



Sources: Allianz Research.

Expressed at a daily rate, the (log) return-to-peak is on average not constant during a bubble. [3]1,040 to 260 trading days before the peak (i.e four to one year before the peak), it increases slowly from 0.15% to 0.25% (or from 37.9% to 87.4% at compounded annual rate); during the last year before the peak, it follows a parabolic path that ends close to 1.5% (equivalent to a 3,866% compounded annual rate).

Our 15% threshold may look somewhat arbitrary. In 1929, the perceived return on the S&P 500 peaked at 12.56% on 03 September. Yet, nine days later, the market peaked and then crashed. In the same vein, the perceived return on the S&P 500 peaked at 12.48% on 25 August 1987, and yet the market crashed in October. Before the Great Financial Crisis, broad indices did not pass our 15% threshold, but some smaller asset classes did: the S&P home builders index in 2005, commodities in 2006-2007, Chinese shares and the FAANGS+ in 2007 (Table 1). However, when we study the distribution of the perceived returns of the 44 markets under investigation, we find that about 90% of our observations are smaller than 15%.

Assuming that past price movements are instrumental in shaping expected returns and thus the market's psychology, we use a little-known algorithm – the Allais (1965, 1966) filter – to account for the way market participants process past returns, memorize them and compute our perceived return (Appendix I). Its only inputs are the daily (log) returns of the market of interest; we assume that its output, the perceived return – a weighted average of past returns, the weighting factors of which are time-varying (or context-dependent) – does measure the

[3] The daily (log) return-to-peak is defined as $((\ln P_0 - \ln P_{-n})/n)$

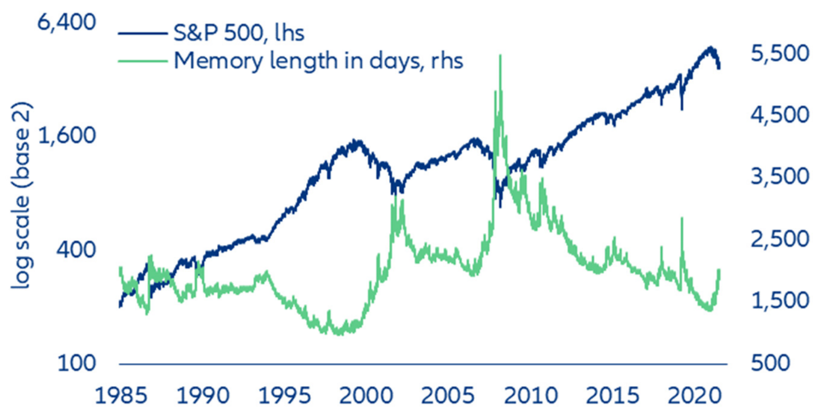
pace at which this market is perceived to rise or to fall. As we compute such perceived return on a daily basis, it is a high-frequency indicator.

Why not use an easier to compute arithmetic average of past daily (log) returns over a fixed timespan? Because, notwithstanding its pervasive use in the financial industry, an equally weighted average of past returns – irrespective of its (arbitrarily chosen) timespan (why one year? why not five?) – ignores the sequential order of returns. Any reshuffling of the sample returns yields indeed the same arithmetic average. As such, an equally weighted average is unlikely to capture the market participants' psychology.

Taking the sequential order of past returns into account is what an exponentially weighted moving average (EWMA) of past returns does. It does so by giving less and less weight to the more and more distant past. Going backward in the past, the weights decline exponentially at a constant rate, set between 0 and 1, called the gain or the decay factor. How should we interpret this constant parameter? In either one of the three following ways: as the speed at which people learn from past experience, as the length of their memory or as an elasticity with respect to the latest outcome. Setting the decay factor close to 1 implies that people learn quickly, have a short memory and are very sensitive to the most recent past. A decay factor close to 0 implies the opposite.

But why should the decay factor be constant? Shouldn't it depend on the context? Increase when the daily returns are in an increasingly steep rising trend, decrease in the opposite situation, in short vary between 0 and 1? By design, the Allais filter's decay factor has such dynamic properties. As the elasticity of the Allais filter is context-dependent (or time-varying), such a smoothing algorithm is uniquely fit to cope with two features of financial time series: on the one hand, the frequent switches between low and high volatility regimes; on the other hand, the parabolic path typically followed by daily returns when a market closes in on a long-lasting high or low. We believe the perceived return generated by the Allais filter is a plausible measure of irrational (i.e. excessive) exuberance (or despair), when people are faced with uncertainty, rather than risk. By uncertainty, we mean a situation in which people do not even know what cannot happen (unknown unknowns). In plain English, Allais's perceived return implies that during a bubble, people give an increasing weight to the most recent past, so that the past becomes increasingly irrelevant (i.e. the (in)famous "this time, it's different" argument) and the length of their memory shrinks (Figure 3). As a result, their expectations become increasingly exuberant, but also increasingly elastic and versatile.

Figure 3: The varying length of memory



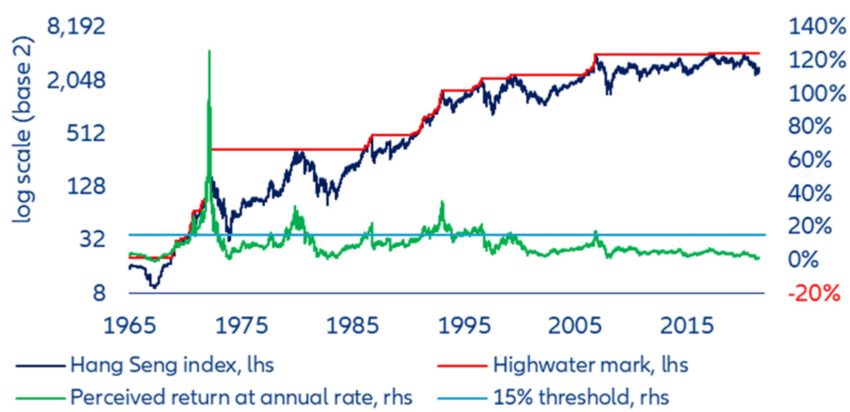
Sources: Refinitiv, Allianz Research.

To define bubbles in a quantitative way and investigate their anatomy, we then combine these two quantitative criteria (Figure 4):

- The high-water mark reached at the peak lasts at least 260 trading days (i.e one year)
- At the high-water mark, the perceived return stands above 15% a year.

The perceived return cannot peak after the market has reached a high-water mark, because the negative returns posted after the peak can only bring it down (see Appendix 1). But, if the market's ascent slows down, the perceived return may peak before the high-water mark is reached (as it did in Japan in the late 1980s).

Figure 4: Combining our two criteria: the Hang Seng S&P example from 1964 to 30 June 2022



Sources: Refinitiv, Allianz Research.

In the case of the Hang Seng index, we identify six bubbles in March 1973, October 1987, January 1994, August 1997, March 2000 and October 2007. For the sake of comparability, we have computed all perceived returns in USD and excluding dividends (for the latter reason, in Germany, we have used the FAZ instead of the DAX index; if the FAZ does not count as a bubble, it is because it narrowly missed our 15% threshold in 1987, 1990, 2000 and 2008).

According to our two criteria, the tally of bubble episodes in equity and commodities markets in the post-WWII era reaches at least 100. Our list of bubbles is most likely not comprehensive because we have by design limited our search to stock market indices and commodities. Except for the FAANGS+, bubbles in specific stocks are therefore out of scope. Bitcoin is shown for the sake of curiosity. The stocks or sectors that led equity markets to record highs in late 2021-early 2022 – the FAANGS, the NASDAQ, the NASDAQ 100, the S&P home builders index – are very likely to increase that number, if they have not already done so (see Table 1).[4] However, the broadest equity market index – the MSCI world – is not as bubbly as it was in 1987 or 2000, but it is as bubbly as in 1973 or 2007.

A remarkable feature of bubbles is visible in Table 1: At the bubble peak, smaller asset classes tend to experience higher perceived returns; in other words, the smaller an asset class, the bigger the bubble. A case in point is the US market in March 2000: According to our perceived return metrics, the Nasdaq biotech index was bubblier than the NASDAQ 100, which was in turn more exuberant than the NASDAQ, which in turn was bubblier than the S&P 500. At the same time, the perceived return on the MSCI World was only 13.2%. The same kind of hierarchy is visible in 1989-90 with the Taiwan SE Taiex, the Kospi, the Nikkei 225 and the MSCI world. And the most extreme perceived returns are associated with "small" asset classes: gold and silver in 1980; bitcoin in 2013 and 2017. From such observations, we can infer that an "everything bubble" is unlikely to ever exhibit perceived returns as high as the ones observed for individual bubbles. The theory of rational bubbles sheds some light on such observations.

The challenge of accounting for the large swings of capital markets without jettisoning the rational expectations hypothesis has led to the concept of rational bubbles: in short, a bubble is deemed rational if is sustainable. According to Jean Tirole (1982, 1985), followed by Olivier Blanchard and Stanley Fischer (1989), under certain conditions, it is rational for prices to deviate from fundamental value, especially if assessing such fundamental value is not straightforward. Which are such conditions? They all relate to the sustainability of the deviation from fundamental value.

- Firstly, current prices must be driven by rational self-confirming expectations about future increases in an asset's price: People must be buying an asset because they expect its price to go up. Under uncertainty, purely rational expectations cannot be formed; expectations à la Allais are more plausible.
- Secondly, for the asset price not to be capped by a final value, the asset must have an infinite life.
- Thirdly, to circumvent the fact that each buyer of the assets has a finite life and must be able to resell his holdings, there must be – like in overlapping generation models – an infinite supply of agents with finite life: the population of potential buyers needs to be constantly rejuvenated.[5]
- Last but not the least, to remain affordable, the bubble's expected rate of return must remain lower than the expected rate of growth of the economy.

---

[4] It is too early to say whether the high-water mark reached in late 2021-early 2022 are long-standing enough (260 trading days) to meet our first criterion.

[5] A Wall Street veteran would probably reformulate this condition by saying there must be an infinite supply of new suckers.

As pointed out by Jean Tirole (2017), it is not easy to apply the theory of rational bubbles to the real world because it involves not directly observable expectations. But its key messages are compatible with the observations we report. A bubble is more likely to inflate when an asset:

- is hard to evaluate (who knows the value of Bitcoin?) and uncertainty rules
- has an infinite life (bonds do not rise as much and as fast as equities)
- appreciates at a slower pace than income (when bubbles burst, the ratio of their market capitalization to aggregate income is typically high)
- and when the population of potential buyers is young and growing (Japan was much younger in the 1980's than it is today).

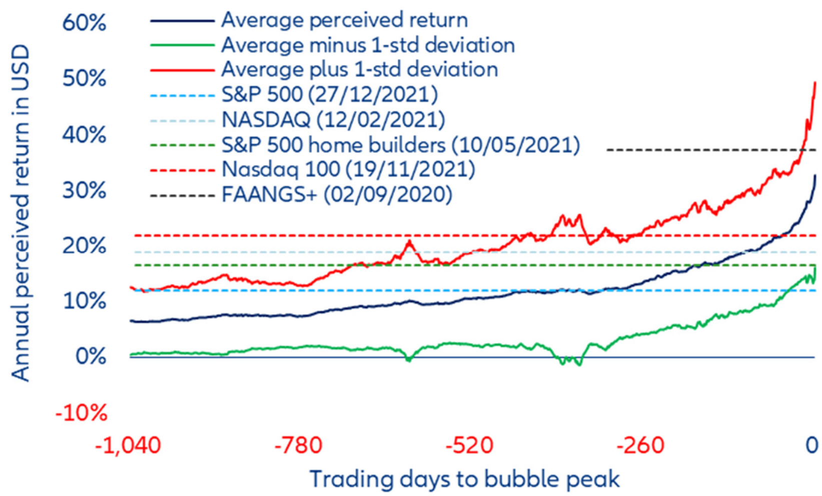
What about fundamentals? An influential branch of academic research suggests that it is hard, if not impossible, to explain the volatility of financial markets in terms of "fundamentals". Building on the works of Irving Fisher (1932) and Hyman Minsky (1986), as well as on his study of financial history, Charles Kindleberger (1978) sees common patterns in different episodes of manias, panics and crashes (see Appendix II). He finds those episodes hard to reconcile with the rational REH. In his view, they provide evidence of a positive feedback loop between price movements and the demand for assets, but he doesn't attempt to quantitatively model non-rational expectations. Robert Shiller (1981) has shown that equity prices move too much to be justified by subsequent changes in dividends. The experimental work of Caginalp, Porter and Vernon Smith (2000) on asset markets shows that people are prone to inflate bubbles, even when they are endowed with perfect information about the "fundamental value" of an asset with a finite life. Vernon Smith explains such observations by the lingering uncertainty about other market participants' behavior, the expectations of which he models with an exponentially weighted moving average (EWMA) of past returns. For Guesnerie (2001, 2005), "it is not obvious to say the least, to explain actual stock markets' fluctuations using dynamic models that adopt some (not too loose) version of the REH". And, according to Phelps and Frydman (2013), "nowhere have the REH's epistemological flaws and empirical disappointments been more apparent than in efforts to model financial market outcomes".

What can we learn from what happened to perceived returns before and after these bubbles burst?

Figure 5 describes the average path of the perceived return (the blue line) in our 100 bubbles.[6] The red and green lines also show the dispersion around the average. Finally, the horizontal dotted lines show the most recent level of the perceived returns on some US indices and the FAANGS+.

[6] The date 0 corresponds to the bubble peak.

Figure 5: The average path followed by the perceived return prior to a bubble peak



Sources: Refinitiv, Allianz Research.

Figure 5 suggests some important observations:

- Four to one year before the peak, the average perceived (log) return increases slowly from 9.4% to 15.7%, a number already much higher than the trend growth rate of nominal GDP and the prevailing rate of interest.
- In the last year before the peak, the average perceived (log) return increases to 30.5% a year (at this rate, prices double every two years and three months).
- In the last year before the peak, the dispersion around the average perceived (log) return increases from 7.9% to 16.7% (an observation already foreshadowed by the dispersion around the average return-to-peak shown in Figure 4)
- Neither the S&P 500 nor the S&P 500 home builders index (a proxy for the US housing market) was in the final year of a typical bubble but – with perceived (log) returns of 11.8% and 14.1% – they were both close to it.
- In contrast, the NASDAQ, the NASDAQ 100 and a capitalization-weighted index of the FAANGS+ (i.e the FAANGS plus Microsoft) – with perceived (log) returns of 18.6%, 21.6% and 25.6%, respectively – might well have been less than six months from their peaks
- Neither Tesla nor Bitcoin can fit in Figure 5 because their perceived returns – 145.6% and 75.3% – are simply off the chart.[7]

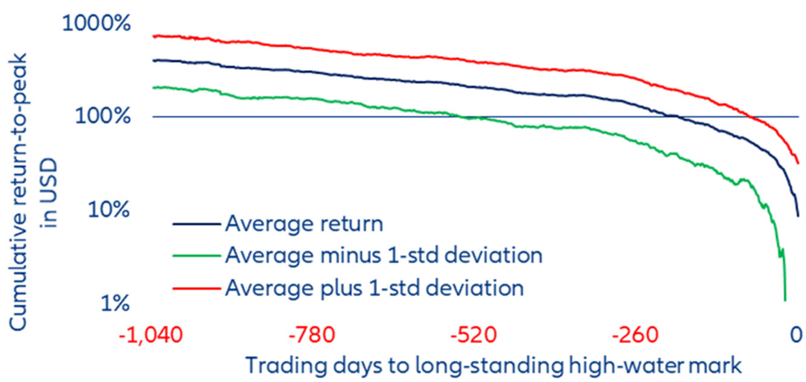
[7] For the sake of brevity, we do not discuss four patterns typically associated with equity bubbles:

- Twin bubbles: a real estate bubble in the same country or equity bubbles in other countries.
- Liquidity: Borrowed or not, abundant liquidity contributes to asset price inflation, but rising collateral values increase in return the supply of loanable funds.
- Leverage: Hidden or not, involving liquidity or not, leverage is another hallmark of equity bubbles.
- New issues of shares: they typically increase during a bubble.

If anything, these four complementary bubble hallmarks were to be seen last year and thus confirm our diagnosis.

Keynes reportedly coined the famous quip according to which "markets can remain irrational for longer than you can stay solvent". Figure 6 illustrates the dilemma facing an investor having qualms about the mindset of other investors: During the last year before the peak, prices on average double (the cumulative return to peak is close to 90%); during the last six months, the average cumulative return is still close to 50%; during the last three months, it still is 35%. In other words, the opportunity cost of an early exit is very high and may represent too high a business risk for the bulk of portfolio managers, tied as they are with benchmarks and relative performances.

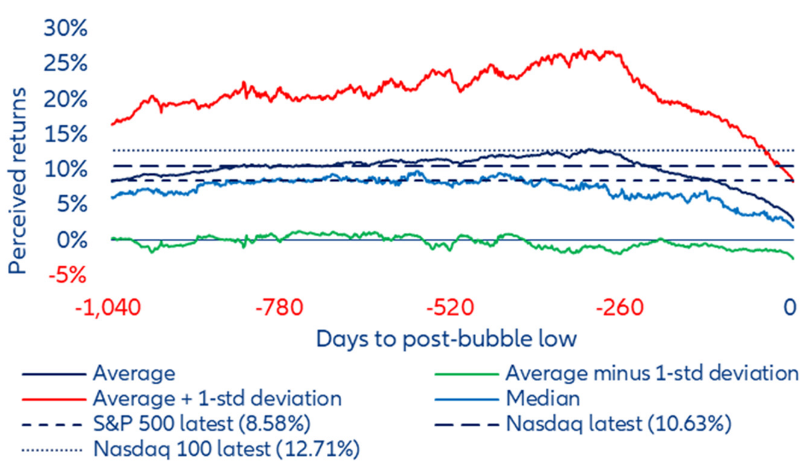
Figure 6: The cost of an early exit



Sources: Refinitiv, Allianz Research.

The typical path followed by the perceived return past a bubble peak suggests that the current correction has still not run its full course. In Figure 7, we plot the average path followed by the perceived return to the post-bubble low (i.e the lowest price level between two consecutive bubbles).[8] In the wake of a bubble burst, a market typically hits a bottom when the perceived return falls close to 2-3%. As of 30 June, the perceived returns of the NASDAQ 100 (10.8%), the NASDAQ (9.1%) and the S&P 500 (7.3%) were still substantially above this level of 2-3%.

Figure 7: The average path followed by the perceived return past a bubble peak

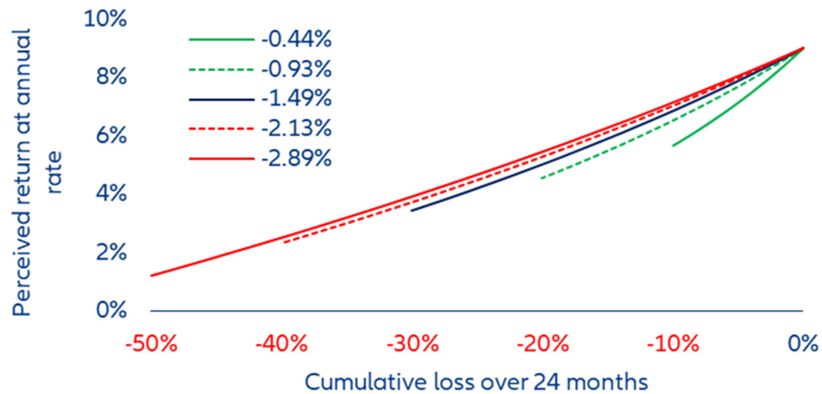


Sources: Refinitiv, Allianz Research.

[8] The date 0 corresponds to the post-bubble low.

It would still take some large price falls to push the perceived return down to the level typically associated with a market trough. To reach this conclusion, we simulate the path followed by the perceived return under different assumptions of average realized monthly return (see Figure 8). A cumulative realized return of only -10% over two years (i.e an average monthly return of -0.44%) would only push the perceived return down from 11% to 7% (see green continuous line). To push it down from 11% to 2% over two years, a cumulative realized return of -50% (i.e an average monthly return of -2.85%) would be needed.

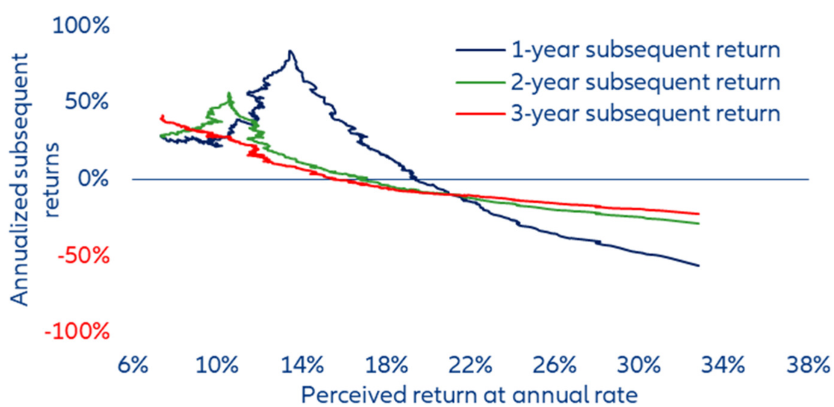
Figure 8: Perceived return simulations



Sources: Refinitiv, Allianz Research.

Such orders of magnitude are consistent with the average return typically observed during the one- to three-year timespan following the time when the perceived return has reached a certain level (Figure 9). Once the perceived return reaches a level of 18-20%, the average annual return during the subsequent two or three years is close to 20-30%.

Figure 9: When a bubble bursts, it hurts

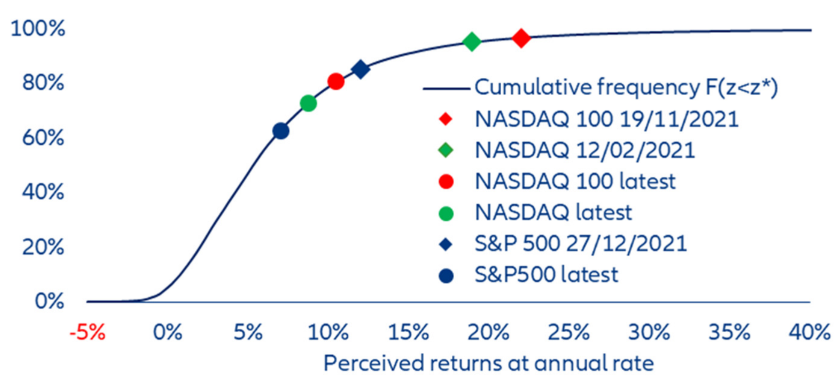


Sources: Refinitiv, Allianz Research.

From a risk management and financial stability point of view, the cumulative distribution of the perceived return indicates whether an asset class entails more downside risk than upside potential. The cumulative distribution of perceived returns shows that some perceived return values are indeed more frequent than others (Figure 10). At each level of the perceived return, one can estimate whether the distribution of potential outcomes is skewed or not.

For example, a little less than 10% of our almost 450,000 daily observations are greater than our 15% criterion. Put differently, there is only one chance out of ten to observe a perceived return higher than 15%, while there are nine chances out of ten of observing a perceived return lower than 15%: the ratio of the upside-to-downside probability is therefore 1-to-9. While a risk-averse decision maker may conclude that such an odds ratio is too low (or that its inverse is too high) to remain (fully) invested (to say the least), a risk-seeking decision maker may still be enticed by the small probability of a large gain. He or she should however be aware that the perceived return being in the end an average, it can only increase if it is fed with fresh outcomes greater than the latest perceived return and therefore ever increasing (see Appendix 1). Under such a scenario, the elasticity of the perceived return would simultaneously rise. As a result, the bubble inflation process would become increasingly challenging and potentially unstable. Such dynamics probably explain why the perceived return tends to peak before prices: a mere slowdown in the pace of the market's ascent is enough to dent the perceived return (as well as market participants' bullishness). Conversely, when the perceived return falls into the low single digit zone, its elasticity is low and increasingly large drawdowns are required to push it further down. This explains why the perceived return falls below 1% so rarely (less than 10% of our observations). As already said, we are not there yet.

Figure 10: Cumulative distribution of the perceived return



Sources: Refinitiv, Allianz Research.

Table 1 – List of bubbles

		Asset class	High-water mark date	Number of days to next high	Perceived log return at high-water mark
1	Japan	Nikkei 225	18 Jul 1961	1,877	23.7
2	Hong Kong	Hang Seng	9 Mar 1973	3,628	122.8
3	Japan	Nikkei 225	15 Mar 1973	1,186	31.1
4	Taiwan	Taiwan SE Taiex	12 Dec 1973	1,223	32.8
5	Commodities	Brent	31 Jan 1974	1,280	32.7
6	Precious metals	Gold	25 Dec 1974	932	17.8
7	Thailand	Bangkok S.E.T	23 Nov 1978	2,222	17.0
8	Commodities	Brent	30 Nov 1979	6,436	35.3
9	Precious metals	Gold	18 Jan 1980	7,252	61.9
10	Precious metals	Silver	21 Jan 1980	8,157	99.8
11	Commodities	Aluminium	29 Feb 1980	2,085	15.2
12	Precious metals	Platinum	5 Mar 1980	6,747	30.3
13	USA	FAANGS+	6 Jun 1983	965	22.0
14	France	CAC 40	7 May 1987	430	20.9
15	UK	FTSE all share	16 Jul 1987	758	20.4
16	Philippines	PSEi	20 Jul 1987	1,596	83.0
17	Netherlands	AEX index	26 Aug 1987	426	16.1
18	World	MSCI world	27 Aug 1987	352	18.0
19	Austria	OBX index	22 Sep 1987	384	15.1
20	USA	FAANGS+	5 Oct 1987	628	35.1
21	Hong Kong	Hang Seng	5 Oct 1987	944	22.3
22	Spain	Madrid SE General	6 Oct 1987	713	29.6
23	Sweden	OMX Stockholm	8 Oct 1987	283	25.2
24	Commodities	Aluminium	31 May 1988	8,747	17.6
25	Finland	OMX Helsinki	5 Dec 1988	1,474	17.0
26	South Korea	Kospi	3 Apr 1989	4,446	38.1
27	Japan	Nikkei 225	27 Dec 1989	8,099	24.9
28	Taiwan	Taiwan SE Taiex	9 Feb 1990	8,026	58.7
29	Austria	OBX index	19 Mar 1990	3,605	42.8
30	France	CAC 40	10 May 1990	526	18.9
31	Spain	Madrid SE General	16 Jul 1990	922	15.7
32	Sweden	OMX Stockholm	31 Jul 1990	1,276	20.1
33	Thailand	Bangkok S.E.T	1 Aug 1990	834	32.5
34	Mexico	Mxipc35	3 Mar 1992	422	41.7
35	India	Nifty 500	27 Apr 1992	3,453	18.6
36	Chile	S&P/CLX IGPA	18 May 1992	417	37.0
37	Thailand	Bangkok S.E.T	4 Jan 1994	>7,287	29.6
38	Hong Kong	Hang Seng	4 Jan 1994	720	35.4
39	Mexico	Mxipc35	8 Feb 1994	2,615	35.6
40	Brazil	Bovespa	12 Sep 1994	478	15.2
41	Peru	S&P BVL General	2 Nov 1994	658	53.2
42	Philippines	PSEi	8 Nov 1994	430	27.5
43	Chile	S&P/CLX IGPA	11 Jul 1995	2,614	31.5
44	Philippines	PSEi	3 Feb 1997	4,087	16.5
45	China	Shangai Real Estate	6 May 1997	2,525	20.8
46	Brazil	Bovespa	8 Jul 1997	2,136	26.4
47	Peru	S&P BVL General	10 Jul 1997	1,751	27.6
48	Hong Kong	Hang Seng	7 Aug 1997	620	22.1
49	Portugal	PSI 20	22 Apr 1998	2,301	22.3
50	Sweden	OMX Stockholm	20 Jul 1998	330	18.8

Table 1 (continued)

		Asset class	High-water mark date	Number of days to next high	Perceived log return at high-water mark
51	Spain	Madrid SE General	5 Jan 1999	295	20.7
52	USA	FAANGS+	22 Dec 1999	1,546	59.5
53	Netherlands	AEX index	3 Jan 2000	1,890	17.5
54	Turkey	Bist National 30	18 Jan 2000	1,570	24.5
55	Spain	Madrid SE General	25 Feb 2000	1,037	16.3
56	USA	Nasdaq biotech	6 Mar 2000	3,390	67.8
57	Finland	OMX Helsinki	6 Mar 2000	1,982	54.4
58	Sweden	OMX Stockholm	6 Mar 2000	1,722	26.0
59	USA	Nasdaq	10 Mar 2000	3,943	42.8
60	USA	S&P 500	24 Mar 2000	1,872	17.0
61	USA	Nasdaq 100	27 Mar 2000	4,070	61.0
62	Hong Kong	Hang Seng	28 Mar 2000	1,716	16.7
63	France	CAC 40	14 Jun 2000	1,510	15.4
64	USA	S&P home builders	20 Jul 2005	3,927	37.5
65	Commodities	Copper	11 May 2006	633	20.4
66	Commodities	Zinc	24 Nov 2006	>3,924	21.1
67	Commodities	Nickel	15 May 2007	>3,802	36.3
68	Sweden	OMX Stockholm	16 Jul 2007	1,635	16.2
69	Peru	S&P BVL General	24 Jul 2007	875	71.2
70	China	Shangai Real Estate	3 Sep 2007	>1974	39.7
71	Commodities	Lead	10 Oct 2007	>3,696	36.6
72	Turkey	Bist National 30	11 Oct 2007	783	20.9
73	China	Shangai A shares	16 Oct 2007	1,993	32.1
74	China	Shenzen Construction	16 Oct 2007	749	27.7
75	Hong Kong	Hang Seng	30 Oct 2007	2,665	17.2
76	Austria	OBX index	31 Oct 2007	>3,826	21.8
77	China	Shenzen Real Estate	1 Nov 2007	1,937	57.0
78	Australia	ASX 200	1 Nov 2007	>3,825	17.3
79	USA	FAANGS+	6 Nov 2007	1,352	30.9
80	Spain	Madrid SE General	8 Nov 2007	>3820	18.3
81	India	Nifty 500	7 Jan 2008	2,557	29.0
82	Precious metals	Platinum	4 Mar 2008	>3,592	19.5
83	Chile	S&P/CLX IGPA	23 Apr 2008	407	15.4
84	Brazil	Bovespa	19 May 2008	>3,561	34.0
85	Norway	OBX index	22 May 2008	2,523	25.2
86	Mexico	Mxipc35	30 May 2008	674	21.1
87	Commodities	Brent	11 Jul 2008	3,499	24.6
88	Chile	S&P/CLX IGPA	3 Jan 2011	>2998	19.1
89	Peru	S&P BVL General	7 Feb 2011	277	27.9
90	Precious metals	Silver	29 Apr 2011	>2,769	30.5
91	Precious metals	Gold	5 Sep 2011	2,318	17.1
92	Peru	S&P BVL General	2 Apr 2012	>2673	21.6
93	Crypto	Bitcoin	29 Nov 2013	840	610.7
94	China	Shangai A shares	12 Jun 2015	>1,694	15.1
95	China	Shangai Real Estate	12 Jun 2015	>1,694	22.9
96	China	Shenzen Real Estate	12 Jun 2015	>1,694	27.2
97	China	Shenzen Construction	12 Jun 2015	>1,694	34.8
98	USA	Nasdaq biotech	20 Jul 2015	1,254	25.2
99	Crypto	Bitcoin	18 Dec 2017	765	490.1

Appendix I – A primer on exponentially-weighted moving average and Allais's tweak

Instead of giving the same weight to all past observations (as in a MAA), a standard EWMA gives them weights that decline exponentially at a constant rate $1 - k$ (the decay factor in a given period equals a constant fraction of the decay factor in the preceding period. The differential definition of the EWMA y of the variable x reads:

$$y_n = y_{n-1} + k(x_n - y_{n-1})$$

with $0 < k \leq 1$, from which we get, by recurrence, the cumulative definition:

$$y_n = \frac{x_n + (1-k)x_{n-1} + (1-k)^2 x_{n-2} + \cdots + (1-k)^n x_0}{1 + (1-k) + (1-k)^2 + \cdots + (1-k)^n} = kY_n$$

with

$$Y_n = x_n + (1-k)x_{n-1} + (1-k)^2 x_{n-2} + \cdots + (1-k)^n x_0$$

and

$$\lim_{n \to +\infty} 1 + (1-k) + (1-k)^2 + \cdots + (1-k)^n = 1/k$$

The characteristic length of such a standard EWMA reads:

$$T = 1/r$$

where $e^{-r} = 1 - k \Leftrightarrow r = -\ln(1-k)$

Since k is constant, T is constant. In a stable environment, in which there is no need to adapt rapidly to fresh information, k can be close to 0. In an unstable one, where it is important to be agile, k should be close to 1. As the parabolic path visible in Figure 2 implies a transition from stability to instability, k instead of being constant, should vary, potentially between 0 and 1.

This is the whole purpose of Allais's tweak: it makes k context-dependent. More precisely, k becomes an increasing non-linear function of Y_{n-1}. Let's call it K_n:

$$z_n = z_{n-1} + K_n(x_n - z_{n-1}) \text{ with } K_n \text{ varying between 0 and 1.}$$

The variation of K through time depends on $Y_{n-1}$and the latest surprise $x_n - z_{n-1}$:

$$dK/dt = f(Y_{n-1}) \times (x_n - z_{n-1})$$

In other words, a sequence of increasing daily returns greater than z_{n-1} causes K to increase and therefore T to decrease. When smoothing the daily returns of an asset in a bubble context, such a feature is quite appealing. For more details, see Barthalon (2014).

Like in a classic EWMA, the perceived return z can increase if and only if the latest outcome x_n is greater than z_{n-1}

$$z_n - z_{n-1} \geq 0 \Leftrightarrow x_n - z_{n-1}$$

Appendix II - The Fisher-Minsky-Kindleberger model of financial instability

- A financial upswing starts for good reasons: (technological) innovation and/or deregulation causes a legitimate increase in the expected return on capital in at least one sector
- As the cost of credit lags behind expected returns on capital employed, the financial upswing is fueled by credit; increased demand for credit is being met by increased supply from banks and/or non-banks
- As the quantity of credit increases, its quality declines: cash-flows initially cover interest and principal payments (hedge finance), then only the former (speculative finance), then neither (Ponzi finance)
- The relaxation of budget constraint brought about by easy credit fosters euphoria, which spreads to other assets or sectors
- Financial leverage inflates profits
- Unusually high asset valuations are being justified by a "new era", not by easy credit
- People succumb to the liquidity illusion, they conflate money velocity (which increases) with their cash ratio (which declines)
- The strength and persistence of the rally fuels adaptive expectations ("the cycle is dead", "this time is different"); the fear of missing out trumps the fear of losing money
- Officials and experts are warning against the risk of excesses, but fail to get attention
- The fear of missing out on another rally makes gullible investors vulnerable to swindles and crime
- Some players, keen on shorting over-valued and over-crowded assets, fail or throw in the towel, as they happen to be “right too early”
- The market finally breaks without obvious reason or fresh information
- The unwinding of leveraged positions leads to distressed selling and forced liquidation; information asymmetry fosters runs on potentially weak intermediaries

Bibliography

Allais, M. (1965), *Reformulation de la théorie quantitative de la monnaie*, SEDEIS, Paris.

Allais, M. (1966), *A restatement of the quantity theory of money*, American Economic Review, Vol. 56, No. 5, pp. 1132-1156.

Barthalon, E. (2014), *Uncertainty, Expectations and Financial Instability - Reviving Allais's Lost Theory of Psychological Time*, Columbia University Press, New-York.

Blanchard, O., Fischer, S. (1989), *Lectures on Macroeconomics*, Chap. 5, *Multiple Equilibria, Bubbles, and Stability*, The MIT Press, Cambridge, Massachusetts.

Caginalp, G., Porter, D., and Smith, V.L. (2000), *Overreactions, momentum, liquidity and price bubbles in laboratory and field markets,* The Journal of Psychology and Financial Markets, 2000, Vol.1, No.1, pp-24-48.

Fisher, I. (1932), *Booms and depressions*, Pickering & Chatto, London, 1996.

Gjertstad, S., Smith, V. (2014), *Rethinking Housing Bubbles - The Role of Household and Bank Balance Sheets in Modeling Economic Cycles*, Chap. 2 *Goods and Services Markets versus Asset Markets*, Cambridge University Press.

Guesnerie, R. (2001, 2005), *Assessing rational expectations*, MIT Press, Cambridge, MA.

Kindleberger, C. (1978), *Manias, Panics and Crashes - A History of Financial Crises*, 4th. ed., Wiley, 2001.

Minsky, H. (1982), *Can "it" happen again?*, Sharpe, New York.

Phelps, E. and Frydman, R. (2013), *Rethinking expectations: The way forward for macroeconomics*, Princeton University Press, Princeton, NJ.

Scheinkman, J. (2014), *Speculation, Trading and Bubbles*, with Kenneth Arrow, Patrick Bolton, Sanford Grossman and Joseph Stiglitz, Columbia University Press, New York.

Shiller, R.J. (1981), *Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?* The American Economic Review, Vol. 71, No. 3 (Jun., 1981), pp. 421-436.

Tirole, J. (1982), *On the Possibility of Speculation under Rational Expectations*, Econometrica, Vol. 50, No. 5, pp.1163-1182.

Tirole, J. (1985), *Asset Bubbles and Overlapping Generations*, Econometrica, Vol. 53, No. 6, pp.1499-1528.

Tirole, J. (2017), *Economics of the Common Good*, Princeton University Press, Princeton.

These assessments are, as always, subject to the disclaimer provided below.